Title	Some Hidden Aspects of Hidden Symmetry
Creators	O'Raifeartaigh, L.
Date	1990
Citation	O'Raifeartaigh, L. (1990) Some Hidden Aspects of Hidden Symmetry. (Preprint)
URL	https://dair.dias.ie/id/eprint/778/
DOI	DIAS-STP-90-46

DIAS-STP-90-46

Some Hidden Aspects of Hidden Symmetry*

L. O'Raifeartaigh

Dublin Institute for Advanced Studies, 10 Burlington Road, Dublin 4, Ireland.

Abstract: It is shown that the Goldstone theorem is actually a special case of the Noether theorem in the presnce of spontaneous symmetry breakdown, and is thus immediately valid for quantized as well as classical fields. The situation when gauge fields are introduced is discussed, emphasis being placed on some points that are not often discussed in the literature such as the compatibility of the Higgs mechanism and the Elitzur theorem and the extent to which the vacuum configuration is determined by the choice of gauge.

1 Introduction.

Although spontaneously-broken gauge theories have been in existence for more than two decades[1], and their experimental relevance has been put beyond all doubt by the theories of (low-temperature) superconductivity[2] and the electroweak interactions[3], new aspects of the theories continue to surface even at the most fundamental and elementary levels. Typical examples are the discovery of the non-trivial topological content of these theories (the existence of monopoles[4], instantons[5] and the Gribov effect[6], for example) and the emergence of paradoxes such as Elitzur's theorem[7], which states that, strictly speaking, gauge symmetries are not spontaneously broken. This means, of course, that, for gauge theories, the name 'spontaneously broken symmetry' is particularly inappropriate, and should be replaced by some alternative such as 'hidden symmetry'. The purpose of the present note is to discuss some of the aspects of hidden gauge symmetries that

---

* Contribution to a Festschrift in Honour of Professor H.-D. Doebner on the occasion of his sixtieth birthday.

still arise, with a view to simplifying and clarifying them. Such a discussion would seem to me to be appropriate in a birthday Festschrift for Professor Doebner since much of his outstanding scientific endeavour has been devoted to simplifying and clarifying the subtler aspects of various theories.

The first, and perhaps most interesting, point to be considered is the intimate connection between the Noether[8] and Goldstone[9] theorems. In the literature these theorems are usually treated as seperate and unrelated, but it is shown in Section 2 that, far from their being unrelated, the Goldstone theorem can actually be derived as a special version of the Noether theorem in the case of a spontaneous breakdown. Some advantages that result from deriving the Goldstone theorem in this way are that it becomes evident at once that the theorem is valid not merely for the minima of the energy (or the Euclidean action) but for any stationary points, and that it does not matter whether the fields are quantized or not[10][11]. All that is actually required is that the condensation point be stationary, which comes out as the absence of 'tadpoles' in the quantized case.

The above discussion is, of course, valid only in the absence of gauge fields, and a question that has to be discussed is the modifications that occur when gauge fields are coupled into the system. The Noether derivation of the Goldstone theorem turns out to be very convenient for purpose. It is first shown (section 3) that the procedure of minimizing the energy (or the Euclidean action) is gauge covariant one in the sense that the vacuum configuration lies on a gauge-invariant orbit. This forms the basis for the Elitzur result, which is essentially the statement that in quantum field theory the functional integration [11] averages over this orbit and so preserves the gauge symmetry.

The most characteristic feature of spontaneously broken gauge theories is, of course, the Higgs mechanism[1][3], through which the Goldstone fields disappear and gauge fields acquire masses. However, since the Goldstone fields disappear only in a particular gauge (the so-called physical one) and the mass-generation is usually computed for a particular vacuum configuration (the constant one in the topologically trivial case) *without* averaging over the orbit, some question arises as to the compatibility of the Higgs mechanism and the Elitzur theorem. This question is discussed for the two parts of the Higgs mechanism, namely the vanishing of the Goldstone fields and the generation of the gauge-field masses, in sections 4 and 5 respectively. In section 4 the Noether derivation of the Goldstone theorem is used to show that, although the Goldstone fields vanish only in a particular gauge, the Goldstone theorem fails in all gauges, and in section 5, it is shown that this the generation of the gauge-field masses is a completely gauge-invariant phenomenon and is thus insensitive to the Elitzur averaging. In both these sections the discussion is not only gauge invariant but also independent of the topology of the underlying configuration space.

Finally, in section 6 we consider a point which arises out of the discussion of sections 4 and 5, but is not often discussed, namely the compatibility of the general gauge choice with the choice of vacuum configuration. We show that, generically, the choice of gauge for general configurations determines the vacuum configuration, but that this is not always the case, and that, in particular, the physical gauge leaves the choice of vacuum configuration completely free.

2 Noether Version of Goldstone Theorem.

We begin by recalling the Noether theorem. Let $L(\eta, \partial_\mu \eta)$, where ∂_μ means $\partial/\partial x_\mu$, be the Lagrangian density for any set of fields $\eta(x)$ and let the Euler-Lagrange field equations for L be written in the form

$$\partial_\mu \pi_\mu = \frac{\partial L}{\partial \eta} \quad , \quad \text{where} \quad \pi_\mu = \frac{\partial L}{\partial \eta_\mu} \; . \tag{1}$$

Now suppose that the fields transform linearly with respect to some x-independent (rigid) continuous group G i.e.

$$\eta(x) \to e^{ia.\sigma} \eta(x) \; , \tag{2}$$

where a_α and σ_α for α = 1,2...dimG, are the (rigid) group parameters and generators respectively. The representation generated by the σs need not, of course, be irreducible, and will certainly not be irreducible if the set of fields $\eta(x)$ includes fields of different spin. Noether's theorem then states that, if one defines the currents

$$j_\mu^\alpha(x) = \sum_\eta \pi_\mu(x) \sigma^\alpha \eta(x) \; , \tag{3}$$

then, as a consequence of the field equations, one has

$$\partial_\mu j_\mu^\alpha = \left(\frac{\delta L(\phi, \partial_\mu \phi)}{\delta a_\alpha} \right)_{a=o} \; , \tag{4}$$

and hence if the Lagrangian density is invariant with respect to the group transformations (2) the currents $j_\mu^\alpha(x)$ are conserved.

In order to derive the Goldstone theorem from this result let us denote the scalar fields in the set $\eta(x)$ by $\phi(x)$ and assume for simplicity that the Lagrangian is of the standard form

$$L(\phi, \partial_\mu\phi, \eta) = \frac{1}{2}(\partial_\mu\phi)^2 + \Lambda(\phi, \eta)\,, \tag{5}$$

where η now denotes all the other fields except the scalar fields and the term $\Lambda(\phi, \eta)$ contains no derivatives of ϕ. Then the Noether currents take the form

$$j^\alpha_\mu(x) = (\partial_\mu\phi(x))\sigma^\alpha\phi(x) + j^\alpha_\mu(\eta(x))\,, \tag{6}$$

where $j^\alpha_\mu(\eta(x))$ denotes the part coming from the fields other than the scalar fields, and may contain the scalar fields but not their derivatives.

Suppose now that the scalar field undergoes a spontaneous symmetry breakdown i.e. the field takes the form

$$\phi(x) = \phi^o + \theta(x)\,, \quad \text{where} \quad \phi^o(x) \neq 0\,, \tag{7}$$

and where ϕ^o, which we shall call the condensate, is a constant stationary point of the action. Then clearly the Noether currents may be written as

$$j^\alpha_\mu = (\partial_\mu\theta(x))\sigma^\alpha\phi^o + j^\alpha_\mu(\theta(x)) + j^\alpha_\mu(\eta(x))\,, \tag{8}$$

and the Noether theorem may be written as

$$\partial_\mu j^\alpha_\mu = \partial^2\theta(x)\sigma^\alpha\phi^o + \partial_\mu j^\alpha_\mu(\theta(x)) + \partial_\mu j^\alpha_\mu(\eta(x)) = 0\,. \tag{9}$$

But since the constant field ϕ^o is supposed to be a stationary point of the action the currents $j_\mu(\theta)$ and $j_\mu(\eta)$ in this equation are bilinears in the fields. Hence equation (9) may be written as

$$\partial^2(\theta(x)\sigma^\alpha\phi^o) = \text{bilinear in the fields}\,. \tag{10}$$

But this shows that the fields $(\theta(x)\sigma^\alpha\phi^o)$ are massless, which is precisely the Goldstone theorem. Of course, if the condensate ϕ^o is only a stationary point but not a minimum of the energy the word word massless is not quite appropriate, since, strictly speaking, masses are defined only at the minima. But the statement that the field equations for the Goldstone fields $(\theta(x)\sigma^\alpha\phi^o)$ contain no linear terms may then be regarded as the generalization of the usual Goldstone statement for arbitrary stationary points. The most interesting point concerning the above derivation, however, is that it makes no explicit distinction between classical and quantized fields. The Noether theorem holds for both kinds of fields, so the only place in which the distinction enters is in the condensate. In the classical case the assumption that the condensate ϕ^o is stationary requires no further consideration after the initial choice, but in the quantized case it requires more careful consideration. In particular, in perturbation theory it requires that the condensate remain stationary in each order of perturbation i.e. it requires that the vacuum be corrected at each order of perturbation so that no tadpole graphs appear. But this is the only distinction between the classical and quantum cases.

It should, perhaps, be emphasized that the question as to whether a spontaneous breakdown actually occurs i.e. whether the field $\phi(x)$ can actually condense to a non-zero constant ϕ^o, lies outside the scope of the above discussion, which is concerned only with the *consequences* of their being a spontaneous breakdown. In particular the well-known statements[12] to the effect that there can be no spontaneous symmetry breakdown in low dimensions are not in contradiction with the above discussion but complementary to it.

3 The Effect of Gauge Fields.

The assumption that the kinetic part of the Lagrangian for the scalar fields is of the form shown in equation (9) eliminates the possibility of the scalar fields being coupled to gauge fields and in this section we wish to consider the modifications that arise if a gauge-fields coupling is introduced. In that case the standard Lagrangian density becomes

$$L(\phi, A_\mu) = \frac{1}{4}\mathrm{tr}F^2 + \frac{1}{2}(D\phi)^2 + \Lambda(\phi) \ , \tag{11}$$

where F is the gauge field, D_μ is the covariant derivative, and, for simplicity, all other fields such as fermion fields have been omitted, so that Λ is actually a potential for ϕ. The field equation for ϕ is still (1) since this was derived for general Lagrangians, but since $\pi_\mu = D_\mu\phi$ is covariant, it is clear that the two sides of (1) are not seperately covariant. It is convenient to make them covariant by adding a term $(A_\mu^\alpha \sigma_\alpha \pi_\mu)$ to each side, in which case the equation takes the form

$$D_\mu \pi_\mu = \frac{\partial \Lambda}{\partial \phi} \ , \tag{12}$$

each side of which is manifestly covariant. The expression (3) for the Noether currents also remains the same, and is covariant, but, by using the field equation (11) one sees that the divergence equation becomes the covariant one

$$D_\mu j^\alpha_\mu = \left(\frac{\delta L}{\delta a_\alpha}\right)_{a=o} . \tag{13}$$

Thus if L is group invariant the covariant divergence vanishes. This is the covariant version of Noethers theorem.

To consider the question of the Goldstone theorem one must now consider the question of a spontaneous symmetry breakdown. The important point to note is that, whereas in the non-gauge case the breakdown is given by equation (7), where the condensate ϕ^o is *constant*, in the gauge case it is given by equation (7) where the condensate ϕ^o is only *covariantly* constant. To see this, we note that the conditions for a the minimum of the energy (or the Euclidean action) for the Lagrangian density (11), denoted by superscript zero, are

$$F^o = 0, \quad D^o\phi^o = 0, \quad \text{and} \quad \Lambda^o = \text{minimum}\,, \tag{14}$$

the first of which implies that the gauge field is pure gauge and the second two imply that $\phi(x)$ lies on gauge invariant group orbit i.e.

$$A^o_\mu = (g^o)^{-1}\partial_\mu g^o \quad \text{and} \quad \phi^o(x) = U(g^o(x))\phi(0)\,, \tag{15}$$

where $x = 0$ is some arbitrary origin in x-space. (Of course, if the x-space is topologically non-trivial $g^o(x)$ may become singular so that (15) holds only in coordinate patches). From (15) one sees that there is no need for $\phi(x)$ to be constant. Indeed it is well-known in monopole and instanton theory that if the x-space is topologically non-trivial and the topological charge is non- zero $\phi(x)$ *cannot* be constant. On the other hand it is clear that $\phi(x)$ lies on a gauge -invariant group orbit and this is the origin of the Elitzur theorem, which is based on the fact that in quantum theory the functional integral averages over the orbit and is thus gauge-invariant. It states essentially that, while the condensate $\phi^o(x)$ in (15) is not zero, the *average* of the condensate with respect to the functional integral (which is equivalent to the vacuum expectation of the field in the canonically quantized version of the theory) is zero. Thus $\phi^o(x) \neq 0$ but $< \phi^o(x) >= 0$.

On the other hand, in the usual treatments of spontaneously broken gauge theories one chooses as vacuum configuration a definite point on the orbit (e.g. ϕ=constant and $A_\mu = 0$ in the topologically trivial case) and this breaks the gauge symmetry. Since this procedure produces the characteristic features of the theory such as the disappearance of the Goldstone fields and the generation of masses for the gauge fields the question then arises as to how these effects can be compatible with the Elitzur result. These are the questions that will be discussed in the following sections.

4 Gauge Analogue of the Noether-Goldstone Theorem.

To obtain the gauge analogue of the Noether-Goldstone theorem we note that, since the Noether currents in the gauge case are of the form

$$j_\mu^\alpha(\phi(x)) = (D_\mu \phi(x))\sigma^\alpha \phi(x) \ , \tag{16}$$

in the case of a spontaneous breakdown they become

$$j_\mu^\alpha(\phi) = (D_\mu \phi^o(x))\sigma^\alpha \phi^o(x) + (D_\mu \phi^o(x))\sigma^\alpha \theta(x) + (D_\mu \theta(x))\sigma^\alpha \phi^o(x) + j_\mu^\alpha(\theta) \ . \tag{17}$$

But since the condensate $\phi^o(x)$ is gauge covariant with respect to the vacuum gauge-field A_μ^o we may write

$$D_\mu \phi^o(x) = (D_\mu - D_\mu^o)\phi^o(x) = \triangle A_\mu(x)\phi^o(x) \quad \text{where} \quad \triangle A_\mu = A_\mu - A_\mu^o \ . \tag{18}$$

and hence

$$\begin{aligned} j_\mu^\alpha(\phi(x)) = {} & \triangle A_\mu^\beta(x)(\sigma_\beta \phi^o(x), \sigma^\alpha \phi^o(x)) + D_\mu(\theta(x), \sigma^\alpha \phi^o(x)) \\ & + 2\triangle A_\mu^\beta(x)(\sigma_\beta \phi^o(x), \sigma^\alpha \theta(x)) + j_\mu^\alpha(\theta(x)) \end{aligned} \ , \tag{19}$$

where the inner product is in the group-representation space, and is used explicitly here and henceforth in order to clarify the notation. Note that $\triangle A_\mu$, being the difference of two connections, is a vector field. Applying the Noether theorem in the covariant form (13) one then obtains as the gauged version of the Goldstone theorem (10) the result

$$D_\mu j_\mu^\alpha(\phi(x)) = D^2(\theta(x), \sigma^\alpha \phi^o(x)) + D_\mu[\triangle A_\mu^\beta(\sigma_\beta \phi^o(x), \sigma^\alpha \phi^o(x))] + \text{bilinears} = 0 \ . \tag{20}$$

Thus even in the gauge theory there is a Goldstone theorem. However, it is only a formal theorem in the sense that it no longer implies the existence of massless fields. To see this, let us write (20) in the non-covariant form

$$\partial^2(\theta(x), \sigma_\alpha \phi^o(x)) + \partial_\mu[\triangle A_\mu^\beta(\sigma_\beta \phi^o(x), \sigma_\alpha \phi^o(x))] = \text{bilinears} \ . \tag{21}$$

From this equation one cannot conclude that the Goldstone field $(\theta(x), \sigma_\alpha \phi^o(x))$ is massless, but only that there is a relationship between its d'Alembertian and the divergence of the vector field $\triangle A_\mu^\alpha(x)$. Thus, in the sense that it predicts the existence of massless fields the Goldstone theorem fails in the presence of gauge fields. This is true in any gauge, but, of course, it is well-known that there exists a gauge, namely the physical gauge, in which it fails even in the stronger sense that the Goldstone fields actually vanish. Since the proof of this is usually only given for the constant vacuum configuration i.e. for $\phi^o(x)$ = constant, it may be worthwhile reproducing it here in a form that is applicable for any vacuum configuration: Consider the functions $f(x,a) = (\theta(x), U(a)\phi^o(x))$, where a^α are the group parameters and $U(a)$ the group-representation to which the scalar fields belong. For each value of x this is a continuous (even analytic) function of the a's and since the range of the a's is compact (assuming the group is compact) it is a function with at least one stationary point. Denoting any stationary point (the minimum, say) for each fixed x by $a^\alpha_s(x)$ we have

$$\left(\frac{\partial f(x,a)}{\partial a_\alpha}\right)_{a=a_s} = v^\alpha_\beta(a_s)(\theta(x), U(a_s)\sigma^\beta\phi^o(x)) = 0 \,, \tag{22}$$

where the $v^\alpha_\beta(a_s)$ is the group velocity matrix. Since this matrix is invertible from the general theory of Lie groups one sees that

$$(\theta(x), U(a_s(x))\sigma_\beta\phi^o(x)) = 0 \,, \tag{23}$$

and hence if we make the gauge transformation

$$\theta(x) \to \theta_g(x), \quad \text{where} \quad \theta_g(x) = U^{-1}(a_s(x))\theta(x) \,. \tag{24}$$

we have

$$(\theta_g(x), \sigma_\alpha\phi^o(x)) = 0 \,. \tag{25}$$

But this shows that the Goldstone fields, which are just the $\sigma_\alpha\phi^o(x)$ components of the scalar fields, vanish in the gauge $\theta = \theta_g$.

5 Covariant Mass Generation.

In this section we wish to show that the mass-generation part of the Higgs mechanism also is a gauge-invariant phenomenon. For this we consider the form that the kinetic term for the scalar fields in the Lagrangian density (11) takes in the case of a spontaneous symmetry breakdown. It is easy to see that the form is

$$L(\phi(x)) = \frac{1}{2}(D_\mu\phi^o(x), D_\mu\phi^o(x)) + (D_\mu\phi^o(x), D_\mu\theta(x)) + L(\theta(x)) \,, \tag{26}$$

which, on using (18), reduces to

$$L(\phi(x)) = \triangle A^{\alpha}_{\mu}\triangle A^{\beta}_{\mu}(\sigma_{\alpha}\phi^{o}(x), \sigma_{\beta}\phi^{o}(x)) + \triangle A^{\alpha}_{\mu}(\sigma_{\alpha}, D_{\mu}\theta(x)) + L(\theta(x)) \ . \quad (27)$$

If we recall that $\triangle A_{\mu}$ is a vector rather than a connection we see that each term in (27) is seperately gauge-invariant. On the other hand, the leading term on the right-hand-side is a mass-term for for this vector field with mass-matrix $M_{\alpha\beta}(x)$ given by $(\sigma_{\alpha}\phi^{o}(x), \sigma_{\beta}\phi^{o}(x))$. Furthermore, since, by definition, we have $M(x) = U^{-1}_{adj}(g(x))M(0)U_{adj}(g(x))$, where $U_{adj}(g)$ denotes the elements of the adjoint representation, the eigenvalues of M(x) are independent of x (even in the topologically non-trivial case) and are therefore genuine physical masses. This generation of physical masses constitutes the covariant version of the Higgs mechanism. It shows that a spontaneously broken potential for the scalar fields will produce masses for the vector fields $\triangle A_{\mu}$ which are gauge-invariant, and therefore quite independent of the Elitzur averaging. Since, from (10), the directions $\sigma_{\alpha}\phi^{o}(x)$ are the Goldstone directions, it is, as usual, just the gauge fields in the Goldstone directions that acquire masses. Note that the mass-generation is not only gauge-invariant but independent of the topology of the x-space and is thus valid even for cases (such as monopole configurations) in which the x-dependence of the scalar condensates $\phi^{o}(x)$ cannot be gauged away.

6 Gauge-Fixing and Vacuum Configurations.

A remarkable feature of the vanishing of the Goldstone fields as described in section 4 is that in the case of the physical gauge it seems to be possible fix the gauge for general configurations and at the same time leave the vacuum configuration arbitrary. To understand why this is remarkable, and is not to be expected for general gauge choices, let us consider the general relationship between the choice of gauge and the choice of vacuum configuration (restricting ourselves, for simplicity, to the topologically trivial case). First we note that, in general, gauge fixing imposes dimG conditions on the fields. On the other hand, the vacuum conditions (15) express all the fields in terms of dimG gauge functions, namely the parameters $a^{o}(x)$ in $g^{o}(x)$=exp$(a^{o}(x)_{\alpha}\sigma^{\alpha})$. Hence a complete gauge fixing would be expected to fix the functions $g^{o}(x)$ uniquely. For example, for scalar QED with one real gauge field $A_{\mu}(x)$ and one complex scalar field $\phi(x)$ the usual gauge chosen is the real gauge, in which the imaginary part of $\phi(x)$ is set equal to zero. Since the scalar-field part of the vacuum conditions in this case are

$$\phi^{o}(x) = \exp(ia^{o}(x))\phi^{o}(0) \ , \quad (28)$$

we see that the real gauge forces the vacuum configuration to be

$$a^o(x) = \text{constant} \quad \Rightarrow \quad \phi^o(x) = \text{constant} \quad \text{and} \quad A^o_\mu(x) = 0 \ . \tag{29}$$

Thus in this case the vacuum configuration is completely fixed by the general gauge fixing, and is the conventional, constant, configuration. Perhaps a more interesting set of gauges (valid for general groups) are the gauges

$$\partial_\mu A^\alpha_\mu(x) = f^\alpha(\phi(x)) \ , \tag{30}$$

where, for example, the functions $f^\alpha(\phi)$ are chosen as

$$f^\alpha(\phi) = 0, \quad f^\alpha(\phi) = (\phi^o, \sigma^\alpha \phi), \quad \text{and} \quad f^\alpha(\phi) = \phi^\alpha, \quad (\phi \in \text{adjoint}) \ , \tag{31}$$

respectively, the first and second being the Landau and 't Hooft gauges, which have the property that they eliminate the bilinear term $\triangle A^\alpha_\mu(\sigma_\alpha, \partial_\mu \theta(x))$ in (24). It is easy to see that for these three gauges the conditions for the vacuum gauge functions $g^o(x)$ are

$$\partial_\mu (g^o(x)^{-1} \partial_\mu g^o(x)) = 0, \quad 0, \quad \text{and} \quad \phi^o(x) \ , \tag{32}$$

respectively, where in the second case we have used the fact that in the adjoint representation the generators are anti-symmetric and in the the third case we have, of course, the relation $\phi^o(x) = (g^o(x))^{-1}\phi^o(0)g^o(x)$. It is clear that in all three cases the vacuum gauge function $g^o(x)$ is determined uniquely up to a function which is a solution of (32) with zero right hand side (and which we shall call a quasi-harmonic function since it reduces to a harmonic function in the abelian case). The lack of complete determination of the vacuum in these cases is, of course, simply due to the fact that the original gauge fixing is itself complete only up to quasi-harmonic functions. From (32) we see that in both the Landau and 't Hooft cases the vacuum is forced to be a trivial configuration i.e. either the constant vacuum configuration or a quasi-harmonic equivalent. But for the third gauge chosen in (31) one sees by inspection that the vacuum configuration *cannot* be the trivial one or a quasi-harmonic equivalent. This is the reason that it was chosen as an example, and is probably one of the reasons that such gauges are not usually considered in the literature!.

Let us now consider the physical gauge in the light of these examples. The condition for the physical gauge is (25). Now, in order to keep the independence of all the scalar fields explicit one must consider them to be real fields, in which case the generators σ are anti-symmetric. On setting $\phi(x)$ equal to $\phi^o(x)$ for the physical gauge and using the anti-symmetry of the σs one sees that the gauge condition (25) is *automatically* satisfied. Thus in the case of the physical gauge the general gauge condition imposes no condition on the $g^o(x)$ and hence no condition on the choice of vacuum configuration (except of course that $g^o(x)$ lie on the group orbit determined by the potential). From the discussion of the other gauges it is clear that in this respect the physical gauge is quite exceptional.

References.

1. See for example: L.Faddeev, A.Slavnov, Gauge Fields, Benjamin/Cummings, New York (1980). T-P. Cheng, L-F Li, Gauge Theory of Elementary Particle Physics, Clarendon Press, Oxford (1984). C. Quigg, Gauge Theories of Strong, Weak and Electromagnetic Interactions, Benjamin/Cummings, New York (1983). C. Lai, R. Mohopatra, Gauge Theories of the Fundamental Interactions, World Scientific, Singapore (1981).
2. A. Fetter, J.Walecka, Quantum Theory of Many Particle Systems, McGraw-Hill, New York (1971).
3. D. Bailin Weak Interactions, Hilger, Bristol (1982). E. Commins, Weak Interactions, McGraw-Hill, New York (1973). C. Lai, Gauge Theory of the Weak and Electromagnetic Interaction, World Scientific, Singapore (1981).
4. N. Craigie, P. Goddard, W. Nahm, Monopoles in Quantum Field Theory, World Scientific, Singapore (1982).
5. S. Coleman, Aspects of Symmetry, Cambridge University Press (1985).
6. P. Ramond, Field Theory, Benjamin/Cummings, New York (1981) page 296, V. Gribov, Nucl. Phys. **B139** (1978) 1.
7. S. Elitzur, Phys. Rev. **D12** (1975) 3978.
8. E. Noether, Gott. Nachr., (1918) 235
9. J. Goldstone, Nuovo Cimento, **19** (1961) 15.
10. J. Goldstone, A. Salam, S. Weinberg, Phys. Rev. **127** (1962) 965.
11. D. Amit, Field Theory, Renormalization Group and Critical Phenomena, McGraw-Hill, New York, (1978).
12. N. Mermin, H. Wagner, Phys. Rev. Lett. **17** (1966) 1133, S. Coleman, Comm. Math. Phys. **31** (1973) 259, S-K. Ma, R. Rajaraman, Phys. Rev. **D11** (1975) 1701.